

## Future Value of an Annuity

An annuity is a series of equal deposits made at equal time intervals. Each deposit is made at the end of each time interval.

### Example # 1:

Ethan deposited \$500 into a savings account at the end of each year. He began on his 13th birthday. The account pays 6%/a. compounded annually. What is the balance in the account on Ethan's 18th birthday.

On Ethan's 18th birthday:

The \$500 deposited at age 13 has earned interest for 5 years.

The amount is  $\$500(1.06)^5$

The \$500 deposited at age 14 has earned interest for 4 years.

The amount is  $\$500(1.06)^4$

etc.....

The amount of Ethan's annuity is the sum of the amounts of the deposits.  
 $= 500(1.06)^5 + 500(1.06)^4 + 500(1.06)^3 + 500(1.06)^2 + 500(1.06)^1 + 500$   
 $= \$3487.66$

The formula for calculating the future value of an annuity is:

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

FV is the future value of the annuity after some time period

R is the **regular** deposit into the investment

i is the adjusted interest rate per compounding period

n is the adjusted number of compounding periods

Example # 2:

Paul deposits <sup>R</sup>\$1000 at the end of each year in an account that pays 5%/a. compounded annually. What is the amount in the account at the end of 10 years?

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$FV = \frac{1000^c [(1 + 0.05^a)^{10^b} - 1]}{0.05^d}$$

$$FV = 12577.89$$

Example # 3:

Annika deposited  $\overset{R}{\$800}$  in an account at the end of every 6 months for  $\overset{n}{7}$  years. The account paid  $\overset{i}{9\%/a.}$  compounded semi-annually. Determine the amount in the account on the date of the last deposit.

$$FV = ?$$

$$R = \$800$$

$$i = \frac{0.09}{2} = 0.045$$

$$n = 7 \times 2 = 14$$

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$FV = \frac{800[(1.045)^{14} - 1]}{0.045}$$

$$FV = 15145.69$$

Homework p. 415 # 1a, 2, 6, 7, 9, 13